1.6. Prüfungsaufgaben zur Potenzrechnung

Aufgabe 1: Potenzen mit natürlichen Exponenten
Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an.
(1. P - 3. P = Potenzregeln, 1. bF – 3. bF = binomische Formeln, D = Definition, K = Kürzen)

a) $2\cdot 2^x$
b) $3^x\cdot 6^x$
c) $\left(\frac{2}{3}\right)^n\cdot 6^n$
d) $10^s : 5^s$
e) $2{,}4^x:(-0{,}8)^x$
f) $4{,}5^k:3^k$
g) $\left(\frac{4}{3}\right)^{12}\cdot\left(\frac{3}{2}\right)^{12}$
h) $(x^2)^3$
i) $(5a^2b^7)^4$
j) $(a^2\cdot b^3)^n$
k) $(x^n\cdot y^m)^{n+m}$
l) $(a^{k-1})^{k+1}$
m) $8^{n+1} - 3\cdot 2^{3n+1}$
n) $\left(\frac{5a^mb^n}{10p^7q^3}\right)^{10}$
o) $\frac{(x^2-4)^3}{(x+2)^3}$
p) $\frac{(x^2-1)^5}{(x-1)^5}$
q) $\frac{a^5\cdot x^3 - a^7\cdot x^5}{a^6\cdot x^4 + a^5\cdot x^3}$
r) $\frac{(a^3+ab^2)^2}{a^6-a^2b^4}$
s) $\frac{(a-b)^{2n}(a^2+ab)^n}{(a^2-b^2)^n a^n}$
t) $\frac{(x^2-y^2)^n x^n}{(x-y)^{2n}(x^2+xy)^n}$

Lösungen

a) $2\cdot 2^x \overset{D}{=} 2^1\cdot 2^x \overset{1.P}{=} 2^{1+x}$ (1)

b) $3^x\cdot 6^x \overset{2.P}{=} (3\cdot 6)^x = 18^x$ (1)

c) $\left(\frac{2}{3}\right)^n\cdot 6^n \overset{2.P}{=} \left(\frac{2}{3}\cdot 6\right)^n = 4^n$ (1)

d) $10^s : 5^s \overset{2.P}{=} (10:5)^s = 2^s$ (1)

e) $2{,}4^x:(-0{,}8)^x \overset{2.P}{=} [2{,}4:(-0{,}8)]^x = [-3]^x$ (1)

f) $4{,}5^k:3^k \overset{2.P}{=} (4{,}5:3)^k = 1{,}5^k$ (1)

g) $\left(\frac{4}{3}\right)^{12}\cdot\left(\frac{3}{2}\right)^{12} \overset{2.P}{=} 2^{12}$ (1)

h) $(x^2)^3 \overset{3.P}{=} x^{2\cdot 3} = x^6$ (1)

i) $(5a^2b^7)^4 \overset{3.P}{=} 5^4\cdot a^{2\cdot 4}\cdot b^{7\cdot 4} = 625a^8b^{28}$ (1)

j) $(a^2\cdot b^3)^n \overset{3.P}{=} a^{2n}\cdot b^{3n}$ (1)

k) $(x^n\cdot y^m)^{n+m} \overset{3.P}{=} x^{n^2+nm}\cdot y^{mn+m^2}$ (1)

l) $(a^{k-1})^{k+1} \overset{3.P}{=} a^{k^2-1}$ (1)

m) $8^{n+1} - 3\cdot 2^{3n+1} = (2^3)^{n+1} - 3\cdot 2^{3n+1} \overset{3.P}{=} 2^{3n+3} - 3\cdot 2^{3n+1} = 2^{3n+1}\cdot(2^2-3) = 2^{3n+1}$. (2)

n) $\left(\frac{5a^mb^n}{10p^7q^3}\right)^{10} \overset{3.P}{=} \frac{a^{10m}b^{10n}}{1024p^{70}q^{30}}$ (1)

o) $\frac{(x^2-4)^3}{(x+2)^3} \overset{3.P}{=} \left(\frac{x^2-4}{x+2}\right)^3 \overset{3.bF}{=} \left(\frac{(x+2)(x-2)}{x+2}\right)^3 = (x-2)^3$ (1)

p) $\frac{(x^2-1)^5}{(x-1)^5} \overset{3.P}{=} \left(\frac{x^2-1}{x-1}\right)^5 \overset{3.bF}{=} \left(\frac{(x+1)(x-1)}{x-1}\right)^5 = (x+1)^5$ (1)

q) $\frac{a^5\cdot x^3 - a^7\cdot x^5}{a^6\cdot x^4 + a^5\cdot x^3} = \frac{a^5x^3(1-a^2x^2)}{a^5x^3(ax+1)} \overset{3.bF}{=} \frac{(1-ax)(1+ax)}{1+ax} = 1 - ax$ (3)

r) $\frac{(a^3+ab^2)^2}{a^6-a^2b^4} \overset{3.bF}{=} \frac{(a^3+ab^2)^2}{(a^3-ab^2)(a^3+ab^2)} = \frac{a^3+ab^2}{a^3-ab^2} \overset{T}{=} \frac{a(a^2+b^2)}{a(a^2-b^2)} = \frac{a^2+b^2}{a^2-b^2}$ (3)

s) $\frac{(a-b)^{2n}(a^2+ab)^n}{(a^2-b^2)^n a^n} \overset{3.P,\ 3.bF}{=} \left(\frac{(a-b)^2 a(a+b)}{(a-b)(a+b)a}\right)^n = (a-b)^n$ (3)

t) $\frac{(x^2-y^2)^n x^n}{(x-y)^{2n}(x^2+xy)^n} \overset{3.P}{=} \left(\frac{(x-y)(x+y)x}{(x-y)^2 x(x+y)}\right)^n = (x-y)^{-n}$ (3)

Aufgabe 2: Potenzen mit ganzzahligen Exponenten
Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrundeliegende Potenzregel an. (1. P - 3. P = Potenzgesetze, 1. – 3- bF = Binomische Formeln, D = Definition)

a) 2^{-3}	f) $\left(\frac{x}{y}\right)^{-n}$	k) $a^2:a^6$	p) $\frac{6ax^{-2}-4x^2}{2x^{-3}}$	u) $(a^4b^{-2} - a^{-2}b^4):(a^2b^{-1} - a^{-1}b^2)$
b) 5^{-2}	g) $\left(\frac{8}{125}\right)^{-1}:\left(\frac{5}{2}\right)^3$	l) $3 : 3^x$	q) $\frac{6xc^{-2}-9cy}{3c^{-5}}$	v) $(x^{-4}y^2 - x^2y^{-4}):(x^{-2}y - xy^{-2})$
c) $\left(\frac{1}{2}\right)^{-3}$	h) $\left(\frac{7}{3^2}\right)^{-2}\cdot\left(\frac{81}{49}\right)^{-2}$	m) $r^{2a}\cdot: r^{a-1}$	r) $\frac{(2xy^2)^2}{(4x^2y^2)^3}:\frac{2y^{-3}}{3x^4}$	w) $\frac{(a^2)^3\cdot(b^3)^{-2}\cdot(ab)^4}{(ab^2)^{-3}\cdot(a^3)^4\cdot b^5}$
d) $\left(\frac{m}{n}\right)^{-k}$	i) $\left(\frac{p}{q}\right)^{-z}:\left(\frac{p}{2q}\right)^{-z}$	n) $x^{1-y} : x^{y-1}$	s) $\frac{(2x^3y^{-2})^4}{(4y)^5}:\frac{(x^4y^{-3})^3}{4y^5}$	x) $(u^{-3} + v^{-3})\cdot(u^{-3} - v^{-3})$
e) $\left(\frac{2}{3}\right)^{-2}$	j) $x^4\cdot x^{-2}$	o) $\frac{x^5\cdot y^{-5}\cdot z^8}{y^{-3}\cdot z^2\cdot x^9}$	t) $\frac{a^6}{b^{m+3}}:\frac{a^8}{b^{m+4}}$	y) $\left(\frac{8c^{-5}}{9a^{-3}b^9}\right)^{-3}\cdot\left(\frac{3a^{-3}c^2}{4b^{-5}}\right)^{-5}$
				z) $\left(\left(1+\frac{2}{p}\right)^2\cdot\left(\frac{1}{p}-\left(\frac{p}{2}-1\right)^{-1}\right)\right)^{-2}$

Lösungen

a) $2^{-3} \overset{D}{=} \frac{1}{2^3} = \frac{1}{8}$ (1)

b) $5^{-2} \overset{D}{=} \frac{1}{5^2} = \frac{1}{25}$ (1)

c) $\left(\frac{1}{2}\right)^{-3} \overset{D}{=} 2^3 = 8$ (1)

d) $\left(\frac{m}{n}\right)^{-k} \overset{D}{=} \left(\frac{n}{m}\right)^{k}$ (1)

e) $\left(\frac{2}{3}\right)^{-2} \overset{D}{=} \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$ (1)

f) $\left(\frac{x}{y}\right)^{-n} \overset{D}{=} \left(\frac{y}{x}\right)^{n} = \frac{y^n}{x^n}$ (1)

g) $\left(\frac{8}{125}\right)^{-1}:\left(\frac{5}{2}\right)^3 \overset{D}{=} \frac{125}{8}:\frac{125}{8} = 1$ (1)

h) $\left(\frac{7}{3^2}\right)^{-2}\cdot\left(\frac{81}{49}\right)^{-2} \overset{2.\,P}{=} \left(\frac{7\cdot 81}{9\cdot 49}\right)^{-2} = \left(\frac{9}{7}\right)^{-2} \overset{D}{=} \frac{49}{81}$ (1)

i) $\left(\frac{p}{q}\right)^{-z}:\left(\frac{p}{2q}\right)^{-z} \overset{D}{=} \left(\frac{q}{p}\right)^{z}\cdot\left(\frac{p}{2q}\right)^{z} \overset{2.\,P}{=} \left(\frac{q}{p}\cdot\frac{p}{2q}\right)^{z} = \left(\frac{1}{2}\right)^{z} \overset{D}{=} 2^{-z}$ (1)

j) $x^4\cdot x^{-2} \overset{1.\,P}{=} x^{4-2} = x^2$ (1)

k) $a^2:a^6 \overset{D,\,1.\,P}{=} a^{2-6} = a^{-4}$ (1)

l) $3 : 3^x \overset{D,\,1.\,P}{=} 3^{1-x}$ (1)

m) $r^{2a}\cdot: r^{a-1} \overset{D,\,1.\,P}{=} r^{2a-(a-1)} = r^{1+a}$ (1)

n) $x^{1-y} : x^{y-1} \overset{D,\,1.\,P}{=} x^{1-y-(y-1)} = x^{2-2y}$ (1)

o) $\frac{x^5\cdot y^{-5}\cdot z^8}{y^{-3}\cdot z^2\cdot x^9} \overset{D,\,1.\,P}{=} x^{5-9}\cdot y^{-5-(-3)}\cdot z^{8-2} = x^{-4}y^{-2}z^6$ (1)

p) $\frac{6ax^{-2}-4x^2}{2x^{-3}} = \frac{6ax^{-2}}{2x^{-3}}-\frac{4x^2}{2x^{-3}} \overset{D,\,1.\,P}{=} 3ax - 2x^5$ (2)

q) $\frac{6xc^{-2}-9cy}{3c^{-5}} = \frac{6xc^{-2}}{3c^{-5}}-\frac{9cy}{3c^{-5}} \overset{D,\,1.\,P}{=} 2xc^3 - 3cy^6$ (2)

r) $\dfrac{(2xy^2)^2}{(4x^2y^2)^3} : \dfrac{2y^{-3}}{3x^4} \overset{\text{3. P}}{=} \dfrac{2^2x^2y^4}{4^3x^6y^6}\cdot\dfrac{3x^4}{2y^{-3}} \overset{\text{1. P}}{=} \dfrac{4\cdot 3\cdot x^{2+4-6}\cdot y^{4-6-(-3)}}{4^3\cdot 2} = \dfrac{3}{32}\,y$ (3)

s) $\dfrac{(2x^3y^{-2})^4}{(4y)^5} : \dfrac{(x^4y^{-3})^3}{4y^5} \overset{\text{3. P}}{=} \dfrac{2^4x^{12}y^{-8}}{4^5y^5}\cdot\dfrac{4y^5}{x^{-12}y^{-9}} \overset{\text{1. P}}{=} \dfrac{2^4\cdot 4\cdot x^{12-12}y^{-8+5-(-9)-5}}{4^5} = \dfrac{1}{16}\,y$ (3)

t) $\dfrac{a^6}{b^{m+3}} : \dfrac{a^8}{b^{m+4}} \overset{\text{D, 1. P}}{=} a^6b^{m+3}a^{-8}b^{-8-(-(m+4))} \overset{\text{1. P}}{=} a^{6-8}b^{m+3+m+4} = a^{-2}b^{2m+7}$ (2)

u) $(a^4b^{-2} - a^{-2}b^4):(a^2b^{-1} - a^{-1}b^2) = \dfrac{\dfrac{a^4}{b^2}-\dfrac{b^4}{a^2}}{\dfrac{a^2}{b}-\dfrac{b^2}{a}} \overset{\text{3.bF}}{=} \dfrac{\left(\dfrac{a^2}{b}-\dfrac{b^2}{a}\right)\cdot\left(\dfrac{a^2}{b}+\dfrac{b^2}{a}\right)}{\dfrac{a^2}{b}-\dfrac{b^2}{a}} = \dfrac{a^2}{b}+\dfrac{b^2}{a}$ (3)

v) $(x^{-4}y^2 - x^2y^{-4}):(x^{-2}y - xy^{-2}) = \dfrac{\dfrac{y^2}{x^4}-\dfrac{x^2}{y^4}}{\dfrac{y^2}{x}-\dfrac{x}{y^2}} \overset{\text{3.bF}}{=} \dfrac{\left(\dfrac{y^2}{x}-\dfrac{x}{y^2}\right)\cdot\left(\dfrac{y^2}{x}+\dfrac{x}{y^2}\right)}{\dfrac{y^2}{x}-\dfrac{x}{y^2}} = \dfrac{y^2}{x}+\dfrac{x}{y^2}$ (3)

w) $\dfrac{(a^2)^3\cdot(b^3)^{-2}\cdot(ab)^4}{(ab^2)^{-3}\cdot(a^3)^4\cdot b^5} \overset{\text{2.+3. P}}{=} \dfrac{a^6\cdot b^{-6}\cdot a^4b^4}{a^{-3}b^{-6}\cdot a^{12}\cdot b^5} \overset{\text{1. P}}{=} a^{6-4-(-3)-12}\cdot b^{-6+4-(-6)-5} = \dfrac{a}{b}$ (3)

x) $(u^{-3} + v^{-3})\cdot(u^{-3} - v^{-3}) \overset{\text{3.bF}}{=} (u^{-3})^2 - (v^{-3})^2 \overset{\text{3. P}}{=} u^{-6} - v^{-6}$ (2)

y) $\left(\dfrac{8c^{-5}}{9a^{-3}b^9}\right)^{-3}\cdot\left(\dfrac{3a^{-3}c^2}{4b^{-5}}\right)^{-5} \overset{\text{3. P}}{=} \dfrac{8^{-3}c^{15}}{9^{-3}a^9b^{-27}}\cdot\dfrac{3^{-5}a^{15}c^{-10}}{4^{-5}b^{25}} \overset{\text{1. P}}{=} \dfrac{2^{3\cdot(-3)}\cdot 3^{-5}\cdot c^{15-10}\cdot a^{15-9}}{3^{2\cdot(-3)}\cdot 2^{2\cdot(-5)}\cdot b^{-27+25}}$ (2)

$\overset{\text{1. P}}{=} \dfrac{2^{-9+10}\cdot 3^{-5+6}\cdot c^5\cdot a^6}{b^{-2}} \overset{\text{1. P}}{=} 6\cdot a^6b^2c^5.$ (2)

z) $\left(1+\dfrac{2}{p}\right)^2\cdot\left(\dfrac{1}{p}-\left(\dfrac{p}{2}-1\right)^{-1}\right)^{-2} = \left(\dfrac{p+2}{p}\right)^2\cdot\left(\dfrac{1}{p}-\left(\dfrac{p-2}{2}\right)^{-1}\right)^{-2}$ (1)

$\overset{\text{3. P}}{=} \left(\dfrac{p+2}{p}\right)^2\cdot\left(\dfrac{1}{p}-\dfrac{2}{p-2}\right)^{-2}$ (1)

$= \left(\dfrac{p+2}{p}\right)^2\cdot\left(\dfrac{p-2-2p}{p\cdot(p-2)}\right)^{-2}$ (1)

$= \left(\dfrac{p+2}{p}\right)^2\cdot\left(\dfrac{p(p-2)}{-p-2}\right)^2$ (1)

$= \left(\dfrac{p+2}{p}\cdot\dfrac{p(p-2)}{(-1)(p+2)}\right)^2 = \left(\dfrac{p-2}{-1}\right)^2 = (p-2)^2.$ (1)

Aufgabe 3: n-te Wurzel
Gib das Ergebnis an und begründe:

a) $\sqrt[4]{256}$ b) $\sqrt[5]{243}$ c) $\left(\sqrt[3]{6}\right)^6$ d) $\left(\sqrt[4]{3}\right)^8$

Lösungen:

$\sqrt[4]{256} = 4$, weil $4^4 = 256$ (2)

$\sqrt[5]{243} = 3$, weil $3^5 = 243$ (2)

$\left(\sqrt[3]{6}\right)^6 \overset{\text{3. P}}{=} \left(\left(\sqrt[3]{6}\right)^3\right)^2 = 6^2 = 36$ (3)

$\left(\sqrt[4]{3}\right)^8 \overset{\text{3. P}}{=} \left(\left(\sqrt[4]{3}\right)^4\right)^2 = 3^2 = 9$ (3)

Aufgabe 4: Potenzen mit rationalen Exponenten: nur 3. Potenzregel/Definition
Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an.
(1. - 3. P = Potenzgesetze, 1. – 3. bF = Binomische Fomeln, D = Definition)

a) $(35^0\cdot 8^{0,5}\cdot 4^1)^2$ f) $25^{\frac{3}{2}}$ k) $\sqrt[5]{2}^{\,-10}$ p) $\sqrt[3]{x^2}^{\,6}$ u) $\left(a^{-\frac{3}{2}}\right)^{-\frac{4}{3}}$

b) $(6^{-4}\cdot 3^{6})^{0,5}$

c) $81^{\frac{1}{4}}$

d) $125^{\frac{1}{3}}$

e) $8^{\frac{2}{3}}$

g) $\left(\frac{1}{\sqrt[3]{5}}\right)^{2}$

h) $\left(\frac{27}{64}\right)^{\frac{2}{3}}$

i) $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

j) $\sqrt[3]{5}^{\;6}$

l) $\sqrt[4]{x}^{\;-2}$

m) $\sqrt[3]{x}^{\;6}$

n) $\sqrt[2]{y^{4}}$

o) $\sqrt{\sqrt[4]{x}}$

q) $\sqrt[5]{y^{3}}^{\;10}$

r) $\sqrt[3]{s^{4}}^{\;3n}$

s) $\sqrt[6]{\sqrt{x}^{\;3}}$

t) $\left(\sqrt{\sqrt{b^{3}}}\right)^{4n}$

v) $\left(x^{-\frac{5}{4}}\cdot y^{-\frac{5}{8}}\right)^{-\frac{4}{5}}$

w) $\sqrt{y\cdot\sqrt{y\cdot\sqrt{y}}}$

x) $\frac{\sqrt{\sqrt[5]{f}}}{\sqrt[5]{f}\cdot\sqrt{f^{3}}}$

Lösungen

a) $(35^{0}\cdot 8^{0,5}\cdot 4^{1})^{2} \overset{D,\,2.\,P}{=} 35^{0}\cdot 8^{1}\cdot 4^{2} \overset{D}{=} 2\cdot 4^{3} = 128$ (1)

b) $(6^{-4}\cdot 3^{6})^{0,5} \overset{D,\,2.\,P}{=} 6^{-2}\cdot 3^{3} \overset{D}{=} \frac{27}{36} = \frac{3}{4}$ (1)

c) $81^{\frac{1}{4}} \overset{D}{=} \sqrt[4]{81} = 3$ (1)

d) $125^{\frac{1}{3}} \overset{D}{=} \sqrt[3]{125} = 5$ (1)

e) $8^{\frac{2}{3}} \overset{3.\,P}{=} \left(8^{\frac{1}{3}}\right)^{2} \overset{D}{=} 2^{2} = 4$ (1)

f) $25^{\frac{3}{2}} \overset{3.\,P}{=} \left(25^{\frac{1}{2}}\right)^{3} \overset{D}{=} 5^{3} = 125$ (1)

g) $\left(\frac{1}{\sqrt[3]{5}}\right)^{2} \overset{D}{=} \left(5^{-\frac{1}{3}}\right)^{2} \overset{3.\,P}{=} 5^{-\frac{1}{3}\cdot 2} \overset{3.\,P}{=} \left(5^{2}\right)^{-\frac{1}{3}} = 25^{-\frac{1}{3}} \overset{D}{=} \frac{1}{\sqrt[3]{25}}$ (1)

h) $\left(\frac{27}{64}\right)^{\frac{2}{3}} \overset{3.\,P}{=} \left(\left(\frac{27}{64}\right)^{\frac{1}{3}}\right)^{2} \overset{D}{=} \left(\frac{3}{4}\right)^{2} = \frac{9}{16}$ (1)

i) $\left(\frac{27}{8}\right)^{-\frac{2}{3}} \overset{D}{=} \left(\frac{8}{27}\right)^{\frac{2}{3}} \overset{3.\,P}{=} \left(\left(\frac{8}{27}\right)^{\frac{1}{3}}\right)^{2} \overset{D}{=} \left(\frac{2}{3}\right)^{2} = \frac{4}{9}$ (1)

j) $\sqrt[3]{5}^{\;6} \overset{D,\,3.\,P}{=} 5^{\frac{6}{3}} = 25$ (1)

k) $\sqrt[5]{2}^{\;-10} \overset{D,\,3.\,P}{=} 2^{-\frac{10}{5}} \overset{D}{=} \frac{1}{4}$ (1)

l) $\sqrt[4]{x}^{\;-2} \overset{D,\,3.\,P}{=} x^{-\frac{2}{4}} \overset{D}{=} \frac{1}{\sqrt{x}}$ (1)

m) $\sqrt[3]{x}^{\;6} \overset{D}{=} \left(x^{\frac{1}{3}}\right)^{6} \overset{3.\,P}{=} x^{\frac{1}{3}\cdot 6} = x^{2}$ (1)

n) $\sqrt[2]{y^{4}} \overset{D}{=} \left(y^{4}\right)^{\frac{1}{2}} \overset{3.\,P}{=} y^{\frac{4}{2}} = y^{2}$ (1)

o) $\sqrt{\sqrt[4]{x}} \overset{D}{=} \left(x^{\frac{1}{4}}\right)^{\frac{1}{2}} \overset{3.\,P}{=} x^{\frac{1}{4}\cdot\frac{1}{2}} \overset{D}{=} \sqrt[8]{x}$ (1)

p) $\sqrt[3]{x^{2}}^{\;6} \overset{D,\,3.\,P}{=} x^{2\cdot\frac{1}{3}\cdot 6} = x^{4}$ (1)

q) $\sqrt[5]{y^{3}}^{\;10} \overset{D,\,3.\,P}{=} y^{3\cdot\frac{1}{5}\cdot 10} = y^{6}$ (1)

r) $\sqrt[3]{s^{4}}^{\;3n} \overset{D,\,3.\,P}{=} s^{4\cdot\frac{1}{3}\cdot 3n} = s^{4n}$ (1)

s) $\sqrt[6]{\sqrt{x}^{\;3}} \overset{D,\,3.\,P}{=} x^{\frac{1}{2}\cdot 3\cdot\frac{1}{6}} = x^{\frac{1}{4}}$ (2)

t) $\left(\sqrt{\sqrt{b^3}}\right)^{4n} \overset{D,\,3.\,P}{=} b^{3\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 4n} = b^{3n}$ (2)

u) $\left(a^{-\frac{3}{2}}\right)^{-\frac{4}{3}} \overset{3.\,P}{=} a^{\left(-\frac{3}{2}\right)\cdot\left(-\frac{4}{3}\right)} = a^2$ (1)

v) $\left(x^{-\frac{5}{4}} \cdot y^{-\frac{5}{8}}\right)^{-\frac{4}{5}} \overset{3.\,P}{=} x^{\left(-\frac{5}{4}\right)\cdot\left(-\frac{4}{5}\right)} \cdot y^{\left(-\frac{5}{8}\right)\cdot\left(-\frac{4}{5}\right)} = xy^{0,5}$ (1)

w) $\sqrt{y\cdot\sqrt{y\cdot\sqrt{y}}} \overset{D}{=} \left(y\cdot\left(y\cdot y^{0,5}\right)^{0,5}\right)^{0,5} \overset{1.\,P}{=} \left(y\cdot\left(y^{1,5}\right)^{0,5}\right)^{0,5} \overset{3.\,P}{=} \left(y\cdot y^{0,75}\right)^{0,5} \overset{1.\,P}{=} \left(y^{1,75}\right)^{0,5} \overset{3.\,P}{=} y^{0,875}$ (4)

x) $\dfrac{\sqrt{\sqrt[5]{f}}}{\sqrt[5]{f}\cdot\sqrt{f^3}} \overset{D}{=} \dfrac{\left(f^{\frac{1}{5}}\right)^{\frac{1}{2}}}{f^{\frac{1}{5}}\cdot f^{\frac{3}{2}}} \overset{3.\,P}{=} \dfrac{f^{\frac{1}{5}\cdot\frac{1}{2}}}{f^{\frac{1}{5}+\frac{3}{2}}} \overset{1.\,P}{=} f^{\frac{1}{10}-\frac{17}{10}} = f^{-\frac{16}{10}} = f^{-\frac{8}{5}}$ (4)

Aufgabe 5: Potenzen mit rationalen Exponenten: nur 1. Potenzregel
Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an.
(1. - 3. P = Potenzgesetze, 1. – 3. bF = binomische Formeln, D = Definition)

a) $3^{\frac{1}{4}}\cdot 9^{\frac{1}{4}}\cdot 3^{\frac{1}{4}}$

b) $2^{\frac{1}{3}}\cdot 4^{\frac{2}{3}}$

c) $5^{\frac{1}{3}}\cdot 25^{\frac{1}{3}}$

d) $a^{\frac{3}{2}}\cdot a^{\frac{5}{2}}$

e) $(a^{\frac{m}{3}}\cdot a^{\frac{m}{6}}) : a^{\frac{m}{4}}$

f) $(3x^{\frac{1}{2}} + 2x^{\frac{2}{3}})\cdot 4x^{\frac{3}{2}}$

g) $3a^{\frac{1}{4}}b^{\frac{2}{3}}\cdot(a-3b^2)$

h) $(6k^{\frac{3}{4}}m^5n^3 + 12k^2m^{\frac{1}{2}}n^{\frac{1}{4}}) : (3k^{-2}m^2n^{\frac{1}{2}})$

i) $\sqrt[3]{2}\cdot\sqrt[3]{4}$

j) $\sqrt[3]{x}\cdot\sqrt{x}$

k) $\dfrac{\sqrt[3]{2y} : \sqrt[3]{y}}{\sqrt{y} : \sqrt[4]{y}}$

l) $\dfrac{\sqrt{t} : \sqrt[3]{t}}{t}$

m) $a\sqrt[3]{b} + b\sqrt[3]{a}\ : \sqrt[3]{ab}$

Lösungen

a) $3^{\frac{1}{4}}\cdot 9^{\frac{1}{4}}\cdot 3^{\frac{1}{4}} = 3^{\frac{1}{4}}\cdot 3^{\frac{2}{4}}\cdot 3^{\frac{1}{4}} \overset{1.\,P}{=} 3^{\frac{1}{4}+\frac{2}{4}+\frac{1}{4}} = 3$ (2)

b) $2^{\frac{1}{3}}\cdot 4^{\frac{2}{3}} = 2^{\frac{1}{3}}\cdot 2^{2\cdot\frac{2}{3}} \overset{1.\,P}{=} 2^{\frac{1}{3}+2\cdot\frac{2}{3}} = 2^{\frac{5}{3}}$ (2)

c) $5^{\frac{1}{3}}\cdot 25^{\frac{1}{3}} = 5^{\frac{1}{3}}\cdot 5^{\frac{2}{3}} \overset{1.\,P}{=} 5^{\frac{1}{3}+\frac{2}{3}} = 5$ (2)

d) $a^{\frac{3}{2}}\cdot a^{\frac{5}{2}} \overset{1.\,P}{=} a^{\frac{3}{2}+\frac{5}{2}} = a^4$ (2)

e) $(a^{\frac{m}{3}}\cdot a^{\frac{m}{6}}) : a^{\frac{m}{4}} \overset{1.\,P}{=} a^{\frac{m}{3}+\frac{m}{6}-\frac{m}{4}} = a^{\frac{m}{4}}$ (2)

f) $(3x^{\frac{1}{2}} + 2x^{\frac{2}{3}})\cdot 4x^{\frac{3}{2}} \overset{1.\,P}{=} 3\cdot 4\cdot x^{\frac{1}{2}+\frac{3}{2}} + 2\cdot 4\cdot x^{\frac{2}{3}+\frac{3}{2}} = 12x^2 + 8x^{\frac{13}{6}}$ (2)

g) $3a^{\frac{1}{4}}b^{\frac{2}{3}}\cdot(a-3b^2) \overset{1.\,P}{=} 3\cdot a^{\frac{1}{4}+1}b^{\frac{2}{3}} - 3\cdot 3\cdot a^{\frac{1}{4}}b^{\frac{2}{3}+2} = 3a^{\frac{5}{4}}b^{\frac{2}{3}} - 9a^{\frac{1}{4}}b^{\frac{8}{3}}$ (2)

h) $(6k^{\frac{3}{4}}m^5n^3 + 12k^2m^{\frac{1}{2}}n^{\frac{1}{4}}) : (3k^{-2}m^2n^{\frac{1}{2}}) \overset{1.\,P}{=} 2k^{\frac{3}{4}+2}m^{5-2}n^{3-\frac{1}{2}} + 4k^{2+2}m^{\frac{1}{2}-2}n^{\frac{1}{4}-\frac{1}{2}} = 2k^{\frac{11}{4}}m^3n^{\frac{5}{2}} + 4k^4m^{-\frac{3}{2}}n^{-\frac{1}{4}}$ (2)

i) $\sqrt[3]{2}\cdot\sqrt[3]{4} \overset{D}{=} 2^{\frac{1}{3}}\cdot 2^{\frac{2}{3}} \overset{1.\,P}{=} 2^{\frac{1}{3}+\frac{2}{3}} = 2$ (2)

j) $\sqrt[3]{x}\cdot\sqrt{x} \overset{1.\,P}{=} x^{\frac{1}{3}}\cdot x^{\frac{1}{2}} \overset{1.\,P}{=} x^{\frac{1}{3}+\frac{1}{2}} = x^{\frac{5}{6}}$ (1)

k) $\dfrac{\sqrt[3]{2y} : \sqrt[3]{y}}{\sqrt{y} : \sqrt[4]{y}} \overset{D,\,1.\,P}{=} 2^{\frac{1}{3}}\cdot y^{\frac{1}{3}-\frac{1}{3}-(\frac{1}{2}-\frac{1}{4})} = 2^{\frac{1}{3}}\cdot y^{-\frac{1}{4}}$ (2)

l) $\dfrac{\sqrt{t} : \sqrt[3]{t}}{t} \overset{D,\,1.\,P}{=} t^{\frac{1}{2}-\frac{1}{3}-1} = t^{-\frac{5}{6}}$ (2)

m) $a\sqrt[3]{b} + b\sqrt[3]{a}\ : \sqrt[3]{ab} \overset{D}{=} \left(ab^{\frac{1}{3}} + a^{\frac{1}{3}}b\right) : a^{\frac{1}{3}}b^{\frac{1}{3}} \overset{1.\,P}{=} a^{1-\frac{1}{3}}b^{\frac{1}{3}-\frac{1}{3}} + a^{\frac{1}{3}-\frac{1}{3}}b^{1-\frac{1}{3}} = a^{\frac{2}{3}} + b^{\frac{2}{3}}$ (2)

Aufgabe 6: Potenzen mit rationalen Exponenten: 1. und 3. Potenzregel
Schreibe als Potenzen mit gleichen Basen und vereinfache:

a) $2\cdot\left(\frac{1}{8}\right)^{\frac{1}{5}}$

b) $ab^2\cdot\left(\frac{4}{a^2b^3}\right)^{\frac{1}{3}}$

c) $6\cdot\left(\frac{7}{36}\right)^{\frac{1}{3}}$

d) $(a+b)\cdot\left(\frac{5}{a+b}\right)^{\frac{1}{2}}$

e) $x\cdot\left(\frac{x}{y}\right)^{\frac{1}{3}}$

f) $(x-y)\cdot\ x^2-y^2\ ^{-\frac{1}{2}}$

g) $\sqrt[2]{a^3}\cdot\sqrt[2]{a^3}$

h) $\sqrt[3]{a^2}\cdot\sqrt[3]{a^2}$

i) $\sqrt[3]{x^2}\cdot\sqrt[3]{x^4}$

j) $\sqrt[5]{a}\cdot\sqrt[5]{a^4}$

k) $\sqrt{\sqrt[3]{16}\cdot\sqrt[9]{64}}$

l) $\sqrt[7]{5^5}:\sqrt[3]{5^2}$

m) $\sqrt[3]{x^4}:\sqrt[4]{x^5}$

n) $\frac{\sqrt{b}\cdot\sqrt[3]{b}}{\sqrt[4]{b^3}}$

o) $\frac{x}{\sqrt[3]{x^2}\cdot\sqrt[4]{x}}$

p) $\frac{\sqrt[6]{a^5}}{\sqrt{a}:\sqrt[3]{a}}$

q) $\sqrt[3]{a^5b}\cdot\sqrt[3]{ab^2}$

Lösungen

a) $2\cdot\left(\frac{1}{8}\right)^{\frac{1}{5}} = 32^{\frac{1}{5}}\cdot\left(\frac{1}{8}\right)^{\frac{1}{5}} = \left(\frac{32}{8}\right)^{\frac{1}{5}} = 4^{\frac{1}{5}}$ (2)

b) $ab^2\cdot\left(\frac{4}{a^2b^3}\right)^{\frac{1}{3}} = a^3b^6\ ^{\frac{1}{3}}\cdot\left(\frac{4}{a^2b^3}\right)^{\frac{1}{3}} = \left(\frac{4a^3b^6}{a^2b^3}\right)^{\frac{1}{3}} = 4ab^3\ ^{\frac{1}{3}}$ (2)

c) $6\cdot\left(\frac{7}{36}\right)^{\frac{1}{3}} = 216^{\frac{1}{3}}\cdot\left(\frac{7}{36}\right)^{\frac{1}{3}} = \left(\frac{216\cdot 7}{36}\right)^{\frac{1}{3}} = 42^{\frac{1}{3}}$ (2)

d) $(a+b)\cdot\left(\frac{5}{a+b}\right)^{\frac{1}{2}} = (a+b)^2\ ^{\frac{1}{2}}\cdot\left(\frac{5}{a+b}\right)^{\frac{1}{2}} = \left(\frac{5(a+b)^2}{a+b}\right)^{\frac{1}{2}} = 5(a+b)\ ^{\frac{1}{2}}$ (2)

e) $x\cdot\left(\frac{x}{y}\right)^{\frac{1}{3}} = x^3\ ^{\frac{1}{3}}\cdot\left(\frac{x}{y}\right)^{\frac{1}{3}} = \left(\frac{x^3\cdot x}{y}\right)^{\frac{1}{3}} = \left(\frac{x^4}{y}\right)^{\frac{1}{3}}$ (2)

f) $(x-y)\cdot\ x^2-y^2\ ^{-\frac{1}{2}} = (x-y)^2\ ^{\frac{1}{2}}\cdot\left(\frac{1}{x^2-y^2}\right)^{\frac{1}{2}} = \left(\frac{(x-y)^2}{(x-y)(x+y)}\right)^{\frac{1}{2}} = \left(\frac{x-y}{x+y}\right)^{\frac{1}{2}}$ (2)

g) $\sqrt[2]{a^3}\cdot\sqrt[2]{a^3} = a^{\frac{3}{2}}\cdot a^{\frac{3}{2}} = a^3$ (1)

h) $\sqrt[3]{a^2}\cdot\sqrt[3]{a^2} = a^{\frac{2}{3}}\cdot a^{\frac{2}{3}} = a^{\frac{4}{3}}$ (1)

i) $\sqrt[3]{x^2}\cdot\sqrt[3]{x^4} = x^{\frac{2}{3}}\cdot x^{\frac{4}{3}} = x^{\frac{2}{3}+\frac{4}{3}} = x^2$ (2)

j) $\sqrt[5]{a}\cdot\sqrt[5]{a^4} = \sqrt[5]{a\cdot a^4} = \sqrt[5]{a^5} = a$ (1)

k) $\sqrt{\sqrt[3]{16}\cdot\sqrt[9]{64}} = \left(4^{2\cdot\frac{1}{3}}\cdot 4^{3\cdot\frac{1}{9}}\right)^{\frac{1}{2}} \overset{(1)}{=} \left(4^{2\cdot\frac{1}{3}+3\cdot\frac{1}{9}}\right)^{\frac{1}{2}} = 4^1\ ^{\frac{1}{2}} = 2$ (1)

l) $\sqrt[7]{5^5}:\sqrt[3]{5^2} = 5^{\frac{5}{7}}\cdot 5^{-\frac{2}{3}} \overset{(1)}{=} 5^{\frac{5}{7}-\frac{2}{3}} = \sqrt[21]{5}$ (1)

m) $\sqrt[3]{x^4}:\sqrt[4]{x^5} = x^{\frac{4}{3}}\cdot x^{-\frac{5}{4}} \overset{(1)}{=} x^{\frac{4}{3}-\frac{5}{4}} = \sqrt[20]{x}$ (1)

n) $\frac{\sqrt{b}\cdot\sqrt[3]{b}}{\sqrt[4]{b^3}} \overset{(1)}{=} b^{\frac{1}{2}+\frac{1}{3}-\frac{3}{4}} = b^{\frac{1}{12}}$ (2)

o) $\frac{x}{\sqrt[3]{x^2}\cdot\sqrt[4]{x}} \overset{(1)}{=} x^{1-\frac{2}{3}-\frac{1}{4}} = x^{\frac{1}{12}}$ (2)

p) $\frac{\sqrt[6]{a^5}}{\sqrt{a}:\sqrt[3]{a}} \overset{(1)}{=} a^{\frac{5}{6}-(\frac{1}{2}-\frac{1}{3})} = a^{\frac{2}{3}}$ (2)

q) $\sqrt[3]{a^5b}\cdot\sqrt[3]{ab^2} = a^5b^1\ ^{\frac{1}{3}}\cdot\ a^1b^2\ ^{\frac{1}{3}} = a^{5+1}b^{1+2}\ ^{\frac{1}{3}} = a^6b^3\ ^{\frac{1}{3}} = a^2b$ (2)

Aufgabe 7: Potenzen mit rationalen Exponenten: 1. - 3. Potenzregel

Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an. (1. – 3-P = Potenzregeln, 1. – 3- bF = binomische Formeln, D = Definition))

a) $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}}$

b) $8^{\frac{1}{2}} : 2^{\frac{1}{2}}$

c) $2^{\frac{3}{2}} \cdot 18^{\frac{3}{2}}$

d) $\sqrt[6]{4a^8b^4} \cdot \sqrt[6]{16a^{16}b^{14}}$

e) $\sqrt[7]{(x^4y)^2z^5} \cdot \sqrt[7]{x^6(y^6z)^2}$

f) $\sqrt{a\sqrt[4]{a}} \cdot \sqrt{a\sqrt{a}}$

g) $\dfrac{1}{a \cdot \sqrt[5]{a^2 \cdot \sqrt{a}}}$

h) $\sqrt[4]{\dfrac{x^8y}{y^{-17}x^6}}$

i) $\sqrt[n]{\dfrac{x^2y^{n+1}}{z^{n+1}}} \cdot \sqrt[n]{\dfrac{zx^{n-2}}{y}}$

j) $\left(\dfrac{a^{-\frac{r}{s}} \cdot b^{\frac{r}{s}}}{a \cdot b^{\frac{r}{s}}}\right)^{-\frac{1}{2}}$

k) $\dfrac{\sqrt[3]{x^2}^{\;2} \cdot x^{\frac{1}{6}}}{\sqrt[5]{x^2} \cdot \sqrt[10]{x}}$

l) $\dfrac{\sqrt[x]{y} \cdot \sqrt[2x]{z^3}}{\sqrt[2x]{y^3} \cdot \sqrt[x]{z}}$

m) $\left(\dfrac{2x^2}{3y^{\frac{3}{8}}}\right) : \left(\dfrac{9x^{-3}}{4y^{\frac{3}{4}}}\right)$

n) $\dfrac{\sqrt[3]{a-b}^{\;4}}{\sqrt[3]{a^2-b^2}^{\;2}}$

o) $\left(x+(x^2-a^2)^{\frac{1}{2}}\right)^3 \cdot \left(x-(x^2-a^2)^{\frac{1}{2}}\right)^3$

p) $\left((x+1)-2x^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot \left((x+1)+2x^{\frac{1}{2}}\right)^{\frac{1}{2}}$

q) $\sqrt[a-b]{x^{a^2-2ab+b^2}} : \sqrt[a+b]{x^{a^2+2ab+b^2}}$

r) $(\sqrt{a}+1)(1-\sqrt[4]{a})(1+\sqrt[4]{a})+a$

e) $(25^{\frac{1}{3}}-4^{\frac{1}{3}}) \cdot (5^{\frac{1}{3}}+2^{\frac{1}{3}})$

Lösungen

a) $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}} \overset{2.P}{=} 27^{\frac{1}{3}} = 3$ (1)

b) $8^{\frac{1}{2}} : 2^{\frac{1}{2}} \overset{2.P}{=} 4^{\frac{1}{2}} = 2$ (1)

c) $2^{\frac{3}{2}} \cdot 18^{\frac{3}{2}} \overset{2.P}{=} 36^{\frac{3}{2}} = 6^3 = 216$ (1)

d) $\sqrt[6]{4a^8b^4} \cdot \sqrt[6]{16a^{16}b^{14}} \overset{1.+2.P}{=} \sqrt[6]{64a^{24}b^{18}} \overset{3.P}{=} 2a^4b^3$ (2)

e) $\sqrt[7]{(x^4y)^2z^5} \cdot \sqrt[7]{x^6(y^6z)^2} \overset{1.+2.P}{=} \sqrt[7]{x^{14}y^{14}z^7} \overset{3.P}{=} x^2y^2z$ (2)

f) $\sqrt{a\sqrt[4]{a}} \cdot \sqrt{a\sqrt{a}} \overset{2.P}{=} \sqrt{a\sqrt[4]{a} \cdot a\sqrt{a}} \overset{D}{=} \left(a^2 \cdot a^{\frac{1}{4}} \cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}} \overset{1.+3.P}{=} a^{\frac{11}{8}}$ (3)

g) $\dfrac{1}{a \cdot \sqrt[5]{a^2 \cdot \sqrt{a}}} \overset{D}{=} a^{-1} \cdot \left(a^2 \cdot a^{\frac{1}{2}}\right)^{-\frac{1}{5}} \overset{1.+3.P}{=} a^{-1} \cdot a^{\frac{5}{2} \cdot \left(-\frac{1}{5}\right)} \overset{1.P}{=} a^{-\frac{3}{2}}$ (3)

h) $\sqrt[4]{\dfrac{x^8y}{y^{-17}x^6}} \overset{1.P}{=} \sqrt[4]{x^2y^{18}} \overset{2.+3.P}{=} x^{\frac{1}{2}} \cdot y^{\frac{9}{2}} \overset{2.+3.P}{=} \sqrt{x \cdot y^9}$ (3)

i) $\sqrt[n]{\dfrac{x^2y^{n+1}}{z^{n+1}}} \cdot \sqrt[n]{\dfrac{zx^{n-2}}{y}} \overset{1.+2.P}{=} \sqrt[n]{\dfrac{x^ny^n}{z^n}} \overset{2.+3.P}{=} \dfrac{xy}{z}$ (3)

j) $\left(\dfrac{a^{-\frac{r}{s}} \cdot b^{\frac{r}{s}}}{a \cdot b^{\frac{r}{s}}}\right)^{-\frac{1}{2}} \overset{1.+3.P}{=} \left(a^{-\frac{r}{s}-\frac{r}{s}} \cdot b^{\frac{r}{s}-\frac{r}{s}}\right)^{-\frac{1}{2}} \overset{3.P}{=} a^{\left(-\frac{r}{s}\right) \cdot \left(-\frac{1}{2}\right)} = a^{\frac{r}{s}}$ (3)

k) $\dfrac{\sqrt[3]{x^2}^{\;2}\cdot x^{\frac{1}{6}}}{\sqrt[5]{x^2}\cdot\sqrt[10]{x}} \overset{D}{=} \dfrac{\left(x^{\frac{2}{3}}\right)^2\cdot x^{\frac{1}{6}}}{x^{\frac{2}{5}}\cdot x^{\frac{1}{10}}} \overset{1.+3.P}{=} x^{\frac{4}{3}+\frac{1}{6}-\frac{2}{5}-\frac{1}{10}} = x$ (3)

l) $\dfrac{\sqrt[x]{y}\cdot\sqrt[2x]{z^3}}{\sqrt[2x]{y^3}\cdot\sqrt[x]{z}} \overset{D}{=} \dfrac{y^{\frac{1}{x}}\cdot z^{\frac{3}{2x}}}{y^{\frac{3}{2x}}\cdot z^{\frac{1}{x}}} \overset{1.P}{=} y^{\frac{1}{x}-\frac{3}{2x}}\cdot z^{-\frac{1}{x}} \overset{3.P}{=} \sqrt[2x]{\dfrac{z}{y}}$ (3)

m) $\left(\dfrac{2x^2}{3y^{\frac{3}{8}}}\right):\left(\dfrac{9x^{-3}}{4y^{\frac{3}{4}}}\right) = \dfrac{2x^2\cdot 4y^{\frac{3}{4}}}{3y^{\frac{3}{8}}\cdot 9x^{-3}} \overset{1.P}{=} \dfrac{8}{27}x^{2+3}y^{\frac{3}{4}-\frac{3}{8}} = \dfrac{8}{27}x^5y^{\frac{3}{8}}$ (2)

n) $\dfrac{\sqrt[3]{(a-b)^4}}{\left(\sqrt[3]{a^2-b^2}\right)^2} \overset{3.P}{=} \dfrac{\sqrt[3]{(a-b)^4}}{\sqrt[3]{(a^2-b^2)^2}} \overset{2.P+3.bF}{=} \sqrt[3]{\dfrac{(a-b)^4}{(a-b)^2(a+b)^2}} = \sqrt[3]{\dfrac{(a-b)^2}{(a+b)^2}} \overset{2.P}{=} \left(\dfrac{a-b}{a+b}\right)^{\frac{2}{3}}$ (3)

o) $\left(x+(x^2-a^2)^{\frac{1}{2}}\right)^3\cdot\left(x-(x^2-a^2)^{\frac{1}{2}}\right)^3 \overset{2.P}{=} \left(\left(x+(x^2-a^2)^{\frac{1}{2}}\right)\cdot\left(x-(x^2-a^2)^{\frac{1}{2}}\right)\right)^3 \overset{2.P}{=} \left(\left(x^2-(x^2-a^2)^{\frac{1}{2}\cdot 2}\right)\right)^3$

$= \quad x^2-(x^2-a^2)\;^3 \overset{3.P}{=} a^6$ (4)

p) $\left((x+1)-2x^{\frac{1}{2}}\right)^{\frac{1}{2}}\cdot\left((x+1)+2x^{\frac{1}{2}}\right)^{\frac{1}{2}} \overset{2.P}{=} \left(\left((x+1)-2x^{\frac{1}{2}}\right)\cdot\left((x+1)+2x^{\frac{1}{2}}\right)\right)^{\frac{1}{2}} \overset{2.P}{=} \left((x+1)^2-\left(2x^{\frac{1}{2}}\right)^2\right)^{\frac{1}{2}}$

$= \quad x^2+2x+1-4x^{\;\frac{1}{2}} = \quad x^2-2x+1^{\;\frac{1}{2}} \overset{2.bF}{=} \quad x-1^{\;2\;\frac{1}{2}} \overset{3.P}{=} x-1$ (5)

q) $\sqrt[a-b]{x^{a^2-2ab+b^2}} : \sqrt[a+b]{x^{a^2+2ab+b^2}} \overset{D+1.+2.bF}{=} \left(x^{(a-b)^2}\right)^{\frac{1}{a-b}} : \left(x^{(a+b)^2}\right)^{\frac{1}{a+b}} \overset{3.P}{=} x^{a-b} : x^{a+b} \overset{1.P}{=} x^{a-b-(a+b)} = x^{-2b}$ (2)

r) $(\sqrt{a}+1)(1-\sqrt[4]{a})(1+\sqrt[4]{a})+a \overset{3.bF+3.P}{=} (\sqrt{a}+1)(1-\sqrt{a})+a \overset{3.bF+3.P}{=} 1-a+a=1$ (2)

s) $(25^{\frac{1}{3}}-4^{\frac{1}{3}})\cdot(5^{\frac{1}{3}}+2^{\frac{1}{3}}) = 25^{\frac{1}{3}}\cdot 5^{\frac{1}{3}}+25^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}-4^{\frac{1}{3}}\cdot 5^{\frac{1}{3}}\cdot -4^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}$

$= 5^{\frac{2}{3}}\cdot 5^{\frac{1}{3}}+25^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}-4^{\frac{1}{3}}\cdot 5^{\frac{1}{3}}\cdot -2^{\frac{2}{3}}\cdot 2^{\frac{1}{3}}$

$\overset{1.+2.P}{=} 5^{\frac{2}{3}+\frac{1}{3}}+(25\cdot 2)^{\frac{1}{3}}-(4\cdot 5)^{\frac{1}{3}}\cdot -2^{\frac{2}{3}+\frac{1}{3}}$

$= 5+50^{\frac{1}{3}}-20^{\frac{1}{3}}-2$

$= 3+50^{\frac{1}{3}}-20^{\frac{1}{3}}$ (3)

Aufgabe 8: Potenzgleichungen

Bestimme die Lösungsmenge der folgenden Gleichungen:

a) $(x-5)^4-81=0$

b) $(x+3)^4-16=0$

c) $(x-2)^4-256=0$

d) $(2x-1)^4=16$

e) $(3x-1)^4=81$

f) $2\cdot(3x-2)^3=4$

g) $3\cdot(2x-3)^5=6$

h) $x^{\frac{3}{4}}=2^{-3}$

i) $\sqrt[6]{x^5}=10^{-5}$

j) $\sqrt[3]{x^2}=2^4$

k) $\sqrt[3]{x^4}=2^{-4}$

l) $\sqrt[3]{x-1}=2$

Lösungen

a) $(x-5)^4 - 81 = 0 \Leftrightarrow x + 5 = \pm 3 \Rightarrow L = \{2; 8\}$ (2)

b) $(x+3)^4 - 16 = 0 \Leftrightarrow x + 3 = \pm 2 \Rightarrow L = \{-1; -5\}$ (2)

c) $(x-2)^4 - 256 = 0 \Leftrightarrow x - 2 = \pm 4 \Rightarrow L = \{-2; 6\}$ (2)

d) $(2x-1)^4 = 16 \Leftrightarrow 2x - 1 = \pm 2 \Rightarrow L = \{-\frac{1}{2}; \frac{3}{2}\}$ (2)

e) $(3x-1)^4 = 81 \Leftrightarrow 3x - 1 = \pm 3 \Rightarrow L = \{-\frac{2}{3}; \frac{4}{3}\}$ (2)

f) $2\cdot(3x-2)^3 = 4 \Leftrightarrow 3x - 2 = \sqrt[3]{2} \Leftrightarrow x = \frac{\sqrt[3]{2}}{3} + \frac{2}{3}$ (2)

g) $3\cdot(2x-3)^5 = 6 \Leftrightarrow (2x-3) = \sqrt[5]{2} \Leftrightarrow x = \frac{\sqrt[2]{5}}{2} + \frac{3}{2}$ (2)

h) $x^{\frac{3}{4}} = 2^{-3} \Leftrightarrow x = {2^{-3}}^{\frac{4}{3}} = 2^{-4} = \frac{1}{16} \Leftrightarrow L = \{\frac{1}{16}\}$ (1)

i) $\sqrt[6]{x^5} = 10^{-5} \Leftrightarrow x^{\frac{5}{6}} = 10^{-5} \Leftrightarrow L = \{10^{-6}\}$ (1)

j) $\sqrt[3]{x^2} = 2^4 \Leftrightarrow x^{\frac{2}{3}} = 2^4 \Leftrightarrow x = 2^{4\cdot\frac{3}{2}} \Leftrightarrow L = \{64\}$ (1)

k) $\sqrt[3]{x^4} = 2^{-4} \Leftrightarrow x^{\frac{4}{3}} = 2^{-4} \Leftrightarrow x = 2^{-4\cdot\frac{3}{4}} \Leftrightarrow L = \{\frac{1}{8}\}$ (1)

l) $\sqrt[3]{x-1} = 2 \Leftrightarrow x - 1 = 2^3 \Rightarrow L = \{9\}$ (1)

Lösung von Exponentialgleichungen durch Exponentenvergleich

Berechne die Lösung der folgenden Gleichungen: $\frac{3^{2x-1}}{27} = 9^{2x+3}$.

a) $3^{x^2-2} = \frac{9}{3^{3x}}$ (3)   b) $\frac{1}{2}2^{x^2-1} = \frac{16}{2^x}$ (3)   c) $\frac{3^{2x-1}}{27} = 9^{2x+3}$ (4)

Lösungen:

a) $3^{x^2-2} = \frac{9}{3^{3x}} \Leftrightarrow 3^{x^2-2} = 3^{2-3x}$ (1)

$\Leftrightarrow x^2 - 2 = 2 - 3x \Leftrightarrow x^2 + 3x - 4 = 0$ (1)

$\Leftrightarrow (x-1)(x+4) = 0 \Leftrightarrow x_1 = 1$ und $x_2 = -4$ (1)

b) $\frac{1}{2}2^{x^2-1} = \frac{16}{2^x} \Leftrightarrow 2^{x^2-2} = 2^{4-x}$ (1)

$\Leftrightarrow x^2 - 2 = 4 - x \Leftrightarrow x^2 + x - 6 = 0$ (1)

$\Leftrightarrow (x-2)(x+3) = 0 \Leftrightarrow x_1 = 2$ und $x_2 = -3$ (1)

c) $\frac{3^{2x-1}}{27} = 9^{2x+3} \Leftrightarrow 3^{2x-1} = 9^{2x+3}\cdot 27 \Leftrightarrow 3^{2x-1} \Leftrightarrow (3^2)^{2x+3}\cdot 3^3$ (1)

$\Leftrightarrow 3^{2x-1} = 3^{2\cdot(2x+3)+3}$ (1)

$\Leftrightarrow 2x - 1 = 2(2x+3) + 3$ (1)

$\Leftrightarrow 2x - 1 = 4x + 9 \Leftrightarrow -10 = 2x \Leftrightarrow x = -5.$ (1)